Compact high-order difference approximations for rod lateral vibrations equation

Vladimir A. Gordin[1], Evgenii A. Tsymbalov[2]

We present a compact high-order difference approximations for rod lateral vibrations:

$$\rho\frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}[R^2\rho\frac{\partial^3 u}{\partial x \partial t^2}] + \frac{\partial^2}{\partial x^2}[ER^2\frac{\partial^2 u}{\partial x^2}] = f,$$

with various boundary conditions. Here, ρ is a density of a rod material, R is rod thickness, E is Young's modulus, f is a right-hand side (forcing). The equation is difficult for calculations because it is not resolved with respect to the highest derivative on time. We consider cases of constant and time-independent variable rod thickness and have studied stability of obtained schemes, as well as different properties such as approximation of first integral and eigenvalues of the boundary problem in various norms: Euclidean, Chebyshev, energetic norm. Numerical experiments show high (4th) order of approximation. Compact difference schemes are effective because they provide high accuracy order for various models of mathematical physics [1-4], and the implicit schemes may be realized by economical double-sweep (Thompson) method.

The report was prepared within the framework of the Academic Fund Program at the National Research University Higher School of Economics (HSE) in 2016–2017 (grant 36STI2015) and supported within the framework of a subsidy granted to the HSE by the Government of the Russian Federation for the implementation of the Global Competitiveness Program.

Literature

1. V.A.Gordin. Mathematics, Computer, Weather Forecasting, and Other Scenarios of Mathematical Physics (in Russian). M., Fizmatlit, 733p, 2010, 2013.
2. V.A. Gordin, E.A.Tsymbalov. Compact difference schemes for the diffusion and Schrodinger equations. Approximation, stability, convergence, effectiveness, monotony // Journal of Computational Mathematics, Vol. 32, No.3, 2014, pp. 348-370.
3. V.A.Gordin, E.A.Tsymbalov. 40th order difference scheme for the differential equation with variable coefficients. Mathematical Simulation, v.29, №7, pp.3-14 (in Russian).
4. V.A.Gordin, E.A.Tsymbalov. Compact difference scheme for the piecewise-constant coefficient. Mathematical Simulation, 2017, v.29, №11, to appear (Russian & English).

[1] Higher School of Economics, Hydrometeorological Center of Russia
[2] Higher School of Economics, Skolkovo Institute of Science and Technology