Stabilization of driven quantum systems moving under the influence of dissipation and noise

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It is well known that a classical particle moving in a viscous medium permanently loses energy stored at the beginning of the motion. To protect the system from the loss of energy, it is necessary to include an external force, the action of which will prevent the action of the friction. For instance, when we are talking about of an optical mode (an electromagnetic field oscillator) localized in the resonator, we have in mind the losses of the energy of the wave in the walls of the resonator. In this case in order to stabilize the mode it is necessary to return back these losses by pumping.

It is clear that without taking into account the fluctuations, the mode can be made stable if one knows the rate of energy loss and replenishes it according to a certain law. In other words, to stabilize the mode, it is necessary to monitor the movement and include an additional loop providing feedback. At the same time, the influence of fluctuations of the bath must be taken into account, since dissipation and fluctuations are connected by the fluctuation-dissipation relation [1], therefore, already at the classical level in the problem of system stabilization, the effects of dissipation and noise cannot be divided.

The situation with the behavior of quantum systems is even more complicated when both dissipation and fluctuations are taken into account. In addition, in quantum systems it is necessary to take into account the fact that monitoring over the system will be accompanied by measurements that introduce additional irreducible effects on the system [2]. Note that recently, in connection with the development of devices for quantum communications, the problem of stabilizing quantum systems is becoming more and more relevant.

In the present work we are going to study a simple quantum system that contains all of the above features. We consider the dissipative dynamics of a quantum oscillator (optical mode) which is placed in a bosonic bath [3]. To control the system, we add an operator that is responsible for the action of the external force. In addition to the energy control problem discussed above, we will also discuss other objective functions that need to be preserved, for example, populations of energy levels and their variances, which can be partially preserved despite the effect of the bath.

The Hamiltonian of the system includes the terms describing the harmonic oscillator (optical mode), the bosonic oscillators (describing the bosonic bath), the interaction of the mode and the bath, and also the term responsible for the nondestructive measurement and the driving force. The method for solving the problem is the following. First of all, the Heisenberg equations of motion for the operators of the system are obtained. The equations for bosonic field operators are linear and may be solved exactly. After eliminating the bosonic operators the remaining equations are solved by approximate and numerical methods. In particular, for the solution of the reduced equation for the density matrix, the quantum Monte Carlo method is used [4,5]. To solve equation numerically we have used Fock basis. The infinite system of equations for mode variables has been truncated and solved by using of the Runge-Kutta method.

The results of numerical simulation, presented in the form of energy and correlation functions dependencies on time, allow predicting the influence of the feedback loop on the behavior of a quantum system.

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