Relaxation Oscillations of the Repressilator Model

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The study of genetic oscillators is of interest, because they are simplified models of key biological processes, such as the cellular cycle and the circadian rhythm. The simplest genetic oscillator called a repressilator consists of three components A_j, $j = 1, 2, 3$. Its distinctive feature is that the component A_3 suppresses the synthesis of A_2, the component A_2 suppresses the synthesis of A_1, and the component A_1, closing the cycle, suppresses the synthesis of A_3. Further each component of the oscillator is a composition of matrix ribonucleic acid (mRNA) of concentration m_j and protein of concentration p_j. The evolution in time of these concentrations is described by the system

$$\dot{m}_j = -m_j + \frac{\alpha}{1 + p_{j+1}^{\gamma}} + \alpha_0,\ \dot{p}_j = \varepsilon(m_j - p_j), \quad j = 1, 2, 3,\ p_4 = p_1, \tag{1}$$

where $\alpha, \alpha_0, \gamma, \varepsilon$ are positive parameters.

As a rule, this model is studied under the assumptions of the smallness of ε and α_0. In this case, replacing $\varepsilon t \to t$ and discarding the parameter α_0, one obtains a singularly perturbed system, to which the well-known Tikhonov reduction principle is applied. The reduction procedure results in the following system:

$$\dot{p}_j = -p_j + \frac{\alpha}{1 + p_{j+1}^{\gamma}}, \quad j = 1, 2, 3, \quad p_4 = p_1. \tag{2}$$

For a fixed $\gamma > 2$ and for $\alpha \ll 1$ we consider the existence, asymptotics, and stability of the self-symmetric cycle (invariant with respect to the cyclic permutation of the coordinates)

$$(p_1, p_2, p_3) = (p(t), p(t + h), p(t + 2h)), \tag{3}$$

where $h > 0$ is the phase shift, and $p(t)$ is periodic with period $T = 3h$.

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