COMPUTATIONAL ASPECTS OF THE WAVE DISTRIBUTION PROBLEM IN THE LOGISTIC EQUATION WITH SPATIAL DEVIATION

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The most common form of logistic equation that accounts the dependency from time and spatial shifts was proposed by Gourley and Britton:

$$\frac{\partial u(t,x)}{\partial t} = \Delta u(t,x) + u(t,x)[1 + \alpha u(t,x) - (1-\alpha)(g * u)(t,x)],$$

where convolution $(g * u)(t,x) = \int_{-\infty}^{t} \int_{\Omega} g(t-\tau, x-y) u(\tau, y) dy d\tau$. If we take delta-function in the integral and move focusing point by time or spatial axis we may get equation with delay or equation with spatial deviation. We considered the process of density wave propagation in a logistic equation with diffusion (Fisher-Kolmogorov equation) and spatial deviation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u\big(1 - u(t, x-y)\big).$$

Here $u(t,x)$ is a solution, which is a population density of FK-problem at time $t > 0$ in some point x of a habitat; $y > 0$ is spatial deviation. A research of qualitative behavior of solutions of Fisher-Kolmogorov equation starts with the analysis of the wave equation profile, for which the conditions of appearance of oscillatory regimes are found. The second problem was the analysis of Fisher-Kolmogorov equation with periodic boundary conditions. We studied the stability loss problem of a spatially homogeneous equilibrium state and found spatially inhomogeneous oscillatory regimes branching from it. The last part of the research contains the results of a large numerical experiment for this equation in the case of an unbounded x domain. Numerical analysis was performed at the cluster of massive and parallel computations with the use of the OpenMP parallel computing technology. A numerical analysis of the wave propagation process showed that for sufficiently small values of the spatial deviation, this equation has solutions close to the solutions of the standard Fisher-Kolmogorov equation. At first the increase of the deviation parameter leads to the appearance of a damped oscillatory component in the spatial distribution of the solution. Further growth of this parameter leads to the destruction of the traveling wave. This is expressed by the fact that undamped in time and propagated slowly along space oscillations are close to the solutions of the corresponding boundary-value problem with periodic boundary conditions in the propagation wave region opposite to the direction of deviation. Finally, if the value of deviation is sufficiently large, then intense spatiotemporal oscillations are observed throughout the wave propagation region. Also we got some interesting analytical results for numerical experiments in case of periodic boundary conditions related with computential artifacts.

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